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BRILLOUIN AND RAMAN SCATTERING OF AN EXTRAORDINARY MODE IN A MAGNETIZED PLASMA

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The effects of magnetic field on the stimulated Brillouin and Raman scattering processes are studied. The formalism applies to plasmas produced by CO_2 lasers and to electron cyclotron heating of toroidal systems by an extraordinary electromagnetic wave. In the case of laser fusion the plasma is magnetized due to the self-generated dc magnetic field while in toroidal plasmas it is due to the external magnetic field. The magnetic field greatly reduces the threshold for Brillouin backscattering by the lower hybrid wave. The Raman scattering by the upper hybrid wave has substantial growth rate even for large $k\lambda_D$ because of the lack of Landau damping.

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I. INTRODUCTION

The absorption of the laser beam by plasma is dependent upon the scattering processes in the underdense region.[1-10] As resonance absorption is one of the major processes by which the laser energy is deposited in the plasma,[11,12] the laser radiation must reach the critical density layer to be effectively absorbed. Hence, it is necessary to know the collective phenomena which will enhance the scattering of the incident radiation and prevent the electromagnetic wave from reaching the critical density.

It has been measured experimentally[13-15] that in laser-pellet experiments there is a self-generated dc magnetic field with its direction perpendicular to the polarization and propagation vectors of the incident laser radiation. Magnetic field of several megagauss has also been predicted theoretically[16-22] and observed in computer simulations.[23] The magnetic field is generated at the critical layer and spreads out to the less dense regions of the plasma by convection and diffusion. Therefore, the investigation of the stimulated scattering processes in a magnetized plasma is a relevant part in the theoretical understanding of the fusion driver-matter interactions.

With the development of high-power cyclotron masers[24] the electron cyclotron heating[25-30] becomes an important plasma heating mechanism of toroidal systems like tokamaks,[31,32] Elmo bumpy torus,[33] etc. If the extraordinary electromagnetic wave is launched into the plasma from the side of the lower magnetic field it needs to tunnel through the evanescent region produced by the cyclotron cutoff in order to be linearly converted into the electron Bernstein wave at the upper hybrid resonance. In addition, the electromagnetic energy can be transferred to the plasma electrons by cyclotron damping at the

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electron cyclotron resonance. However, if the extraordinary electromagnetic wave is injected into the plasma in the direction of increasing major radius the cyclotron cutoff region is avoided.

Similar to the problem found in laser fusion, the stimulated scattering of the incident electromagnetic wave may prevent the energy from reaching the upper hybrid and electron cyclotron resonance layers. Therefore, in the present paper we also examine the role played by the stimulated Brillouin and Raman scattering instabilities in reflecting the incident microwave energy launched into the toroidal plasmas for the purpose of supplementing the initial Joule heating.

For the stimulated Brillouin scattering in a magnetized plasma, the enhanced scattering is due to the excitation of an electrostatic lower hybrid wave instead of the usual ion acoustic wave in an unmagnetized plasma, while, in the stimulated Raman scattering it is due to the upper hybrid wave instead of Langmuir waves.

In Sec. II, the general formalism of nonlinear scattering processes in a magnetized plasma is developed. In Sec. III, the growth rates for the stimulated Brillouin and Raman scattering are found with applications to fusion produced by CO_2 lasers and electron cyclotron heating in toroidal devices. Thresholds and the effects of collisionless damping on the Raman instability are discussed in Sec. IV. In Sec. V, the discussion and conclusions are presented.

II. GENERAL FORMALISM

Consider an extraordinary electromagnetic pump wave $\vec{E}_0=(E_{0x}\hat{x}+E_{0y}\hat{y})\times \cos(k_0x-\omega_0t)$ incident on a plasma with the static magnetic field B_0^0 along the z direction. The Lorentz force drives the electrons with density oscillations

$$n_0 = \frac{n_0^0 \vec{k}_0 \cdot \vec{V}_0}{\omega_0} \tag{1}$$

where n_0^0 is the number density of the unperturbed electrons and $\vec{V}_0$ is the pump wave induced velocity of the electrons whose components are given by

$$V_{0x} = -i \frac{e}{m} \frac{\omega_0 E_{0x} - i\Omega_e E_{0y}}{\omega_0^2 - \Omega_e^2} \tag{2}$$

and

$$V_{0y} = -i \frac{e}{m} \frac{\omega_0 E_{0y} + i\Omega_e E_{0x}}{\omega_0^2 - \Omega_e^2} \quad . \tag{3}$$

The magnetic field of this pump wave is described by

$$\vec{B}_0 = \frac{c}{\omega_0} \vec{k}_0 \times \vec{E}_0 \tag{4}$$

whereupon the electrostatic component of the electric field is

$$E_{0x} = -i \frac{\Omega_e}{\omega_0} \frac{\omega_p^2}{\omega_0^2 - \omega_{uh}^2} E_{0y} \tag{5}$$

where

$$\Omega_e = \frac{eB_0^0}{mc}, \quad \omega_p^2 = \frac{4\pi n_0^0 e^2}{m}, \quad \omega_{uh}^2 = \omega_p^2 + \Omega_e^2 ,$$

and e,m being the electron charge and mass, respectively, c is the speed of light.

The purpose of the present work is to study the interaction of this pump wave with a scattered electromagnetic wave and an electrostatic decay wave, whose Fourier components are $(\vec{k}_- = \vec{k} - \vec{k}_0,\ \omega_- = \omega - \omega_0)$ and $(\vec{k}, \omega)$, respectively. The electromagnetic waves are subjected only to a collisional damping which is weak and can be neglected. Although fluid equations will be used in the present formalism, the collisionless damping of the waves can be taken into account phenomenologically.

The equation for the electrostatic wave is determined by using Poisson's equation

$$\nabla^2\phi = 4\pi e(n-n_i) \tag{6}$$

where ϕ is the perturbed electrostatic potential and, n and n_i are the perturbed electron and ion densities, respectively. The wave equation for the electromagnetic decay wave is found from Maxwell's equation as

$$\nabla^2\vec{E}_- - \vec{\nabla}(\vec{\nabla}\cdot\vec{E}_-) + \frac{\omega_-^2}{c^2}\overset{\leftrightarrow}{\epsilon}_{L_-}\cdot\vec{E}_- = -\frac{4\pi i\omega_-}{c^2}\vec{J}_-^{NL} \tag{7}$$

where $\vec{E}_-$ is the perturbed electric field and $\vec{J}_-^{NL}$ is the nonlinear part of the perturbed current density originating from the heating of the oscillating velocity in the incident electromagnetic wave with the low frequency density oscillations

$$\vec{J}_- = \vec{J}_-^L + \vec{J}_-^{NL} = -en_0^0\vec{V}_- - \frac{1}{2}en_0^*\vec{V} - \frac{1}{2}en\vec{V}_0^* \ . \tag{8}$$

The anti-Stokes component was neglected as being off-resonant. The dielectric tensor $\overset{\leftrightarrow}{\epsilon}_{L_-}$ is defined as

$$\overset{\leftrightarrow}{\epsilon}_{L_-} \equiv \overset{\leftrightarrow}{I} + \frac{4\pi i}{\omega_-}\overset{\leftrightarrow}{\sigma}_- \tag{9}$$

where $\overset{\leftrightarrow}{I}$ is the unity tensor and $\overset{\leftrightarrow}{\sigma}_-$ is the conductivity tensor related to the linear current density by

$$\vec{J}_-^L = \overset{\leftrightarrow}{\sigma}_-\cdot\vec{E}_- \ . \tag{10}$$

The perturbed density oscillation n and velocities $\vec{V}$ and $\vec{V}_-$ are determined from the continuity equation and the equations of motion, respectively:

$$\frac{\partial n}{\partial t} + \vec{\nabla}\cdot(n_0^0\vec{V} + \frac{1}{2}n_0\vec{V}_- + \frac{1}{2}n_-\vec{V}_0) = 0 \ , \tag{11}$$

$$\frac{\partial \vec{V}}{\partial t} + \frac{1}{2}\vec{V}_0\cdot\vec{\nabla}\vec{V}_- + \frac{1}{2}\vec{V}_-\cdot\vec{\nabla}\vec{V}_0 = \frac{e}{m}\vec{\nabla}\phi - \vec{V}\times\vec{\Omega}_e$$
$$- \frac{V_e^2}{n_0^0}\vec{\nabla}n - \frac{e}{2mc}\left(\vec{V}_-\times\vec{B}_0 + \vec{V}_0\times\vec{B}_-\right) , \tag{12}$$

and

$$\frac{\partial \vec{V}_-}{\partial t} + \frac{1}{2}\vec{V}_0^*\cdot\vec{\nabla}\vec{V} + \frac{1}{2}\vec{V}\cdot\vec{\nabla}\vec{V}_0^* = -\frac{e\vec{E}_-}{m}$$
$$-\vec{V}_-\times\vec{\Omega}_e - \frac{e}{2mc}\vec{V}\times\vec{B}_0^* \tag{13}$$

where V_e^2=T/m, T being the electron thermal energy, and $\vec{B}_-$ is the perturbed magnetic field of the scattered wave. The perturbed electron density can be separated into the linear and nonlinear parts

$$n = n^L + n^{NL} \tag{14}$$

where

$$n^L = -\frac{\dfrac{en_0^0\phi}{m}\left(k_{\parallel}^2 + \dfrac{\omega^2}{\omega^2-\Omega_e^2}k_{\perp}^2\right)}{\omega^2 - \left(k_{\parallel}^2 + \dfrac{\omega^2}{\omega^2-\Omega_e^2}k_{\perp}^2\right)V_e^2} , \tag{15}$$

$k_{\parallel}$ and $k_{\perp}$ designating the parallel and perpendicular components of $\vec{k}$ in relation to the static magnetic field, and n^{NL} is found from Eqs. (11)-(13). The ions are considered cold and unmagnetized. The nonlinear effects such as the ponderomotive force on the ions can be neglected as it is smaller by a mass ratio to that of electrons. The ion perturbed density is given by

$$n_i = \frac{en_0^0k^2\phi}{M\omega^2} \tag{16}$$

where M is the ion mass. The substitution of Eqs. (14)-(16) into Eq. (6) gives

$$\varepsilon_L\phi = -\frac{4\pi e}{k^2}n^{NL} \tag{17}$$

where ε_L is the dispersion function

$$\varepsilon_L = 1 - \frac{\omega_{pi}^2}{\omega^2} - \frac{\omega_p^2\left(\frac{k_\parallel^2}{k^2} + \frac{\omega^2}{\omega^2-\Omega_e^2}\frac{k_\perp^2}{k^2}\right)}{\omega^2 - \left(k_\parallel^2 + \frac{\omega^2}{\omega^2-\Omega_e^2} k_\perp^2\right)V_e^2} \quad . \tag{18}$$

Equation (15) can be rewritten as

$$\varepsilon_L k\phi = \vec{\alpha}^T \cdot \vec{E}_- \tag{19}$$

where $\vec{\alpha}^T$ is expressed by the transpose of matrix $\vec{\alpha}$ describing the low frequency beating of the two electromagnetic waves. After Fourier analyzing Eq. (7), we get

$$\overleftrightarrow{D}\cdot\vec{E}_- = \overleftrightarrow{\beta}\cdot(\vec{k}\phi) \tag{20}$$

where $\overleftrightarrow{D}$ is the usual dispersion tensor

$$\overleftrightarrow{D} = -k_-^2\overleftrightarrow{I} + \vec{k}_-\vec{k}_- + \frac{\omega_-^2}{c^2}\overleftrightarrow{\varepsilon}_{L_-} \tag{21}$$

and $\overleftrightarrow{\beta}$ is expressed by the matrix containing the coupling between the pump wave and the low frequency electrostatic wave. The components of the dielectric tensor are found from Eqs. (9)-(13) to be

$$\overleftrightarrow{\varepsilon}_{L_-} = \begin{pmatrix} 1 - \frac{\omega_p^2}{\omega_-^2-\Omega_e^2} & i\,\frac{\Omega_e}{\omega_-}\frac{\omega_p^2}{\omega_-^2-\Omega_e^2} & 0 \\ -i\,\frac{\Omega_e}{\omega_-}\frac{\omega_p^2}{\omega_-^2-\Omega_e^2} & 1 - \frac{\omega_p^2}{\omega_-^2-\Omega_e^2} & 0 \\ 0 & 0 & 1 - \frac{\omega_p^2}{\omega_-^2} \end{pmatrix} \quad . \tag{22}$$

Equations (19) and (20) comprise the set of coupled equations describing the parametric processes under investigation.

In the following we shall discuss the scattering in the plane perpendicular to the magnetic field which is also the maximum growing modes for backscattering. In this case $k_{\parallel}=0$ and Eq. (18) reduces to

$$\varepsilon_L = 1 - \frac{\omega_{pi}^2}{\omega^2} - \frac{\omega_p^2}{\omega^2 - \Omega_e^2 - k^2 V_e^2} \tag{23}$$

and the nonvanishing components of $\vec{\alpha}^T$ are

$$\alpha_x = \frac{\omega_p^2 \Omega_e^2}{2(\omega_-^2 - \Omega_e^2)(\omega^2 - \Omega_e^2 - k^2 V_e^2)} \left\{ \frac{i k_0 V_{0x}}{\omega_0} \left(\frac{\omega_0 (k\omega_- + k_- \omega)}{k_0 \Omega_e^2} + \frac{\omega_- (\omega^2 - \Omega_e^2)}{\omega \Omega_e^2} \right) + \frac{\omega_- k_0 V_{0y}}{\omega \Omega_e} + \frac{i e E_{0y} k_0 (\omega + \omega_-)}{m \omega_0 \omega \Omega_e} \right\} \tag{24}$$

and

$$\alpha_y = \frac{\omega_p^2 \Omega_e^2}{2(\omega_-^2 - \Omega_e^2)(\omega^2 - \Omega_e^2 - k^2 V_e^2)} \left\{ \frac{k_0 V_{0x}}{\omega_0} \left(\frac{\omega_0 (k\omega_- + k_- \omega)}{k_0 \omega_- \Omega_e} + \frac{\omega^2 - \Omega_e^2}{\omega \Omega_e} \right) + \frac{i k_0 V_{0y}}{\omega_0} \left(- \frac{\omega_0}{\omega} + \frac{k_- \omega_0 (\omega_-^2 - \Omega_e^2)}{k_0 \omega_- \Omega_e^2} \right) + \frac{e E_{0y} k_0}{m \omega_0^2} \left(\frac{\omega_0}{\omega} + \frac{\omega_- \omega_0}{\Omega_e^2} \right) \right\} . \tag{25}$$

From Eq. (21) the components of the dispersion tensor become

$$\vec{\vec{D}} = \begin{pmatrix} \frac{\omega_-^2}{c^2}\left(1 - \frac{\omega_p^2}{\omega_-^2 - \Omega_e^2}\right) & i \frac{\omega_-^2}{c^2} \frac{\Omega_e}{\omega_-} \frac{\omega_p^2}{\omega_-^2 - \Omega_e^2} & 0 \\ -i \frac{\omega_-^2}{c^2} \frac{\Omega_e}{\omega_-} \frac{\omega_p^2}{\omega_-^2 - \Omega_e^2} & \frac{\omega_-^2}{c^2}\left(1 - \frac{\omega_p^2}{\omega_-^2 - \Omega_e^2}\right) - k_-^2 & 0 \\ 0 & 0 & \frac{\omega_-^2}{c^2}\left(1 - \frac{\omega_p^2}{\omega_-^2}\right) - k_-^2 \end{pmatrix} \tag{26}$$

and the relevant components of the coupling tensor $\overleftrightarrow{\beta}$ are

$$\beta_{xx} = - \frac{\omega_p^2}{2c^2(\omega_-^2-\Omega_e^2)(\omega^2-\Omega_e^2-k^2V_e^2)} \Bigg\{ \frac{ik_0V_{0x}^*}{\omega_0} \left[\frac{\omega_0\omega_-^2}{k_0} (k\omega_- + k_-\omega) + \omega\omega_-(\omega_-^2-\Omega_e^2) \right] - k_0V_{0y}^*\Omega_e\omega\omega_- + \frac{ieE_{0y}^*}{m\omega_0} k_0\Omega_e\omega_-(\omega+\omega_-) \Bigg\} \tag{27}$$

and

$$\beta_{yx} = \frac{\omega_p^2}{2c^2(\omega_-^2-\Omega_e^2)(\omega^2-\Omega_e^2-k^2V_e^2)} \Bigg\{ \frac{k_0V_{0x}^*}{\omega_0} \left[\frac{\omega_0\omega_-\Omega_e}{k_0} (k\omega_- + k_-\omega) + \Omega_e\omega_-(\omega_-^2-\Omega_e^2) \right] + \frac{ik_0V_{0y}^*}{\omega_0} \left[\omega_0\omega\omega_-^2 - \frac{k\omega_0\omega_-(\omega_-^2-\Omega_e^2)}{k_0} \right] + \frac{ek_0E_{0y}^*}{m\omega_0} \omega_-(\omega_-\omega+\Omega_e^2) \Bigg\} \; . \tag{28}$$

Expressions (24)-(28) reduce to those of unmagnetized plasma if we set $\Omega_e=0$.

From Eq. (20) we obtain

$$\vec{E}_- = \overleftrightarrow{D}^{-1} \cdot [\overleftrightarrow{\beta} \cdot (\vec{k}\phi)] = \frac{Ad_j \overleftrightarrow{D}^{-1}}{||D||} \cdot [\overleftrightarrow{\beta} \cdot (\vec{k}\phi)] \tag{29}$$

where $||D||$ represents the determinant of the matrix $\overleftrightarrow{D}$ and $Ad_j\overleftrightarrow{D}^{-1}$ denotes the adjunt of the inverse to the matrix $\overleftrightarrow{D}$. Substituting Eq. (29) into Eq. (19) we get the expression for the dispersion relation

$$\epsilon_L \, ||D_1|| = \vec{\alpha}^T \cdot [Ad_j \overleftrightarrow{D}_1^{-1} \cdot (\overleftrightarrow{\beta} \cdot \hat{k})] \tag{30}$$

where

$$\overleftrightarrow{D}_1^{-1} = \frac{\overleftrightarrow{D}^{-1}}{D_{zz}} = \begin{pmatrix} D_{yy} & 0 & 0 \\ 0 & D_{xx} & 0 \\ 0 & 0 & \dfrac{D_{xx}D_{yy}-D_{xy}D_{yx}}{D_{zz}} \end{pmatrix}, \tag{31}$$

$$\|D_1\| = D_{xx}D_{yy}-D_{xy}D_{yx}, \tag{32}$$

and $\hat{k}=\vec{k}/k$. The dispersion relation (30) is valid for the scattered electromagnetic wave making an arbitrary angle with the incident wave. It also applies if the electrostatic wave is either an eigenmode or a quasimode. If we assume resonant scattering by eigenmode, i.e., $\varepsilon_L(\vec{k},\omega)\simeq 0$, we can expand ε_L and $\|D_1\|$ about the eigenfrequencies $\omega_{\vec{k}}[\varepsilon_L(\omega_{\vec{k}})=0]$ and $\omega_{\vec{k}_-}$ $[\|D_1\|(\omega_{\vec{k}_-})=0]$ to get the growth rate

$$\gamma^2 = -\frac{\vec{\alpha}^T\cdot[\mathrm{Adj}\overleftrightarrow{D}_1^{-1}\cdot(\overleftrightarrow{\beta}\cdot\hat{k})]}{\left.\dfrac{\partial\varepsilon_L}{\partial\omega}\right|_{\omega_{\vec{k}}}\left.\dfrac{\partial\|D_1\|}{\partial\omega_-}\right|_{\omega_{\vec{k}_-}}}. \tag{33}$$

For the backscattering process, expression (33) reduces to

$$\gamma^2 = -\frac{D_{yy}\alpha_x\beta_{xx}+D_{xx}\alpha_y\beta_{yx}}{\left.\dfrac{\partial\varepsilon_L}{\partial\omega}\right|_{\omega_{\vec{k}}}\left.\dfrac{\partial\|D_1\|}{\partial\omega_-}\right|_{\omega_{\vec{k}_-}}} \tag{34}$$

where

$$D_{xx} = \frac{\omega_-^2}{c^2}\left(1-\frac{\omega_p^2}{\omega_-^2-\Omega_e^2}\right),\quad D_{yy} = \frac{\omega_-^2}{c^2}\left(1-\frac{\omega_p^2}{\omega_-^2-\Omega_e^2}\right)-k_-^2, \tag{35}$$

$$\left.\frac{\partial\varepsilon_L}{\partial\omega}\right|_{\omega_{\vec{k}}} = \frac{2\omega_{pi}^2}{\omega^3}+\frac{2\omega\omega_p^2}{(\omega^2-\Omega_e^2-k^2V_e^2)^2}, \tag{36}$$

and

$$\left.\frac{\partial\|D_1\|}{\partial\omega_-}\right|_{\omega_{\vec{k}_-}} = \frac{2\omega_-^3}{c^4}\,\frac{(\omega_-^2-\omega_{uh}^2)^2+\omega_p^2\Omega_e^2}{(\omega_-^2-\Omega_e^2)(\omega_-^2-\omega_{uh}^2)}. \tag{37}$$

For a scattering process in the plane perpendicular to the magnetic field $k_{\parallel}=0$, J_{-z}^{NL} vanishes and, hence, $\beta_{zx}\equiv 0$. Consequently, the decay of an extraordinary electromagnetic wave into an electrostatic and an ordinary electromagnetic waves is not possible.

For a nonresonant scattering by the quasimode, the growth rate is given by

$$\gamma = -\frac{i\vec{\alpha}^T\cdot[\mathrm{Adj}\overleftrightarrow{D}_1^{-1}\cdot(\overleftrightarrow{\beta}\cdot\hat{k})]}{\varepsilon_L \left.\frac{\partial\,\|D_1\|}{\partial\omega_-}\right|_{\omega_{\vec{k}_-}}} . \tag{38}$$

For the backscattering case this expression reduces to

$$\gamma = -\frac{i(D_{yy}\alpha_x\beta_{xx}+D_{xx}\alpha_y\beta_{yx})}{\varepsilon_L \left.\frac{\partial\,\|D_1\|}{\partial\omega_-}\right|_{\omega_{\vec{k}_-}}} . \tag{39}$$

III. BRILLOUIN AND RAMAN SCATTERING INSTABILITIES

A. Laser Fusion

The validity of this investigation is limited by the fluid assumption which for a magnetized plasma requires that $2k_0\rho_e \lesssim 0.5$ ($\rho_e = V_e/\Omega_e$). For instance, for a CO_2 laser ($k_0=5.9\times10^3\,\mathrm{cm}^{-1}$), a self-generated magnetic field of 2 Mgauss ($\Omega_e=3.5\times10^{13}$ rad/s), and plasma temperature of 1 keV ($V_e=1.3\times10^9$ cm/s), then, $2k_0\rho_e\approx 0.4$ and the theory is only valid marginally for these parameters. Obviously, for CO_2 lasers and stronger magnetic fields or colder plasmas the condition of validity of the theory is well satisfied. In the case of Nd-glass laser ($k_0=5.9\times10^4\,\mathrm{cm}^{-1}$), for a 4 Mgauss magnetic field ($\Omega_e=7\times10^{13}$ rad/s), the theory is valid only in a very cold plasma with maximum plasma

temperature of 50 eV ($V_e=3\times10^8$ cm/s). We conclude that kinetic theory should be used to describe, more realistically, plasmas produced by Nd-glass lasers where the temperatures are the order of a few keV. In the kinetic formalism, it is possible to study the scattering by Bernstein modes which might give quite different results. This remains to be investigated.

To study the stimulated Brillouin backscattering in a magnetized plasma in the limit valid for laser fusion, $\omega_p>>\Omega_e$, the low frequency normal mode is obtained from Eq. (23) as $\omega\simeq(\Omega_e\Omega_i)^{1/2}$. By reducing the expressions (24), (25), (27), (28), and (35)-(37) to this limit and substituting into Eq. (34), the growth rate becomes

$$\gamma_B = \frac{1}{2} k_0 V_0 \left(\frac{\omega_{pi}^2}{\omega_0\Omega_e}\right)^{1/2} \left(\frac{M}{m}\right)^{1/4} . \tag{40}$$

The growth rate in an unmagnetized plasma[4] is given by

$$\gamma = \frac{1}{2} k_0 V_0 \left(\frac{\omega_{pi}^2}{\omega_0 k c_s}\right)^{1/2} . \tag{41}$$

The ratio between these two growth rates is given by

$$\frac{\gamma_B}{\gamma} = (k\rho_e)^{1/2} . \tag{42}$$

For instance, if $\omega_0=1.78\times10^{14}$ rad/s (CO_2 laser), $B_0^0=2$ Mgauss, and $T_e=1$ keV, then $\gamma_B/\gamma\simeq0.66$.

For the stimulated Raman instability, Eq. (23) reduces to $\omega=(\omega_p^2+\Omega_e^2)^{1/2}$. From Eq. (34) the growth rate is found to be

$$\gamma_R = \frac{1}{2} k_0 V_0 \left(\frac{\omega_p^2}{\omega_0\omega_{uh}}\right)^{1/2} . \tag{43}$$

The growth rate for the Raman scattering in an unmagnetized plasma[4] is

$$\gamma = \frac{1}{2} k_0 V_0 \left(\frac{\omega_p}{\omega_0}\right)^{1/2} . \tag{44}$$

Then,

$$\frac{\gamma_R}{\gamma} = \left(\frac{\omega_p}{\omega_{uh}}\right)^{1/2} \tag{45}$$

which is close to one for laser fusion parameters.

B. Electron Cyclotron Heating

The condition $2k_0\rho_e \lesssim 0.5$ for the validity of the present formalism is well verified for microwave injection in toroidal plasmas in the direction of increasing major radius. The magnitude of k_0 increases from zero at the upper hybrid cutoff to the vacuum value ω_0/c at the plasma boundary. Supposedly, we want to deposit the wave energy in the central part of the system, then, the upper hybrid resonance should be as close as possible to the center of the plasma and that is achieved by choosing

$$\omega_0 = \omega_{uh} = [\omega_p^2(r=0) + \Omega_e^2(r=0)]^{1/2} \tag{46}$$

where r is the variable which describes the minor radius of the toroidal device. If we assume $\omega_p(r=0)/\Omega_e(r=0) \simeq 1$, then Eq. (46) gives $\omega_0 \simeq \sqrt{2}\Omega_e(r=0)$. The toroidal magnetic field in tokamaks decreases as 1/R when R increases where R describes the major radius, then, the maximum value of the electron cyclotron frequency in the region where the electromagnetic wave propagates occurs just before the upper hybrid cutoff layer. If we assume a parabolic density profile to determine the cutoff layer, we find that $\Omega_e(r=\text{cutoff}) \simeq 0.9\ \Omega_e\ (r=0)$ and, then, $\rho_e \simeq V_e/0.9\ \Omega_e\ (r=0)$. We conclude that

$$2k_0\rho_e = \frac{2\sqrt{2}\Omega_e(r=0)}{c}\ \frac{V_e}{0.9\ \Omega_e(r=0)} \simeq \frac{3V_e}{c} << 1\ . \tag{47}$$

For microwave injection from the inner part of the torus, the theory is not valid for Brillouin and Raman scattering processes near the upper hybrid resonance layer since the refraction index becomes infinity when $\omega_0 \simeq (\omega_p^2+\Omega_e^2)^{1/2}$. However, in practice we like also to have the wave energy transferred to resonant electrons by cyclotron damping if we inject the microwave with finite $k_{\parallel}$. In this case, $\omega_0 = \Omega_e + k_{\parallel} V_{\parallel}$ and the electromagnetic wave reaches the cyclotron resonance layer before reaching the upper hybrid resonance. Then, for $\omega_0 = \Omega_e \simeq \omega_p$, $k_0 \simeq \omega_0/c \simeq \Omega_e/c$, and we have

$$2k_0\rho_e = 2\,\frac{\Omega_e}{c}\,\frac{V_e}{\Omega_e} \simeq \frac{2V_e}{c} << 1. \tag{48}$$

To investigate the Brillouin backscattering process in the limit $\omega_p \simeq \Omega_e$, valid for electron cyclotron heating of toroidal plasmas, the frequency of the electrostatic wave obtained from Eq. (23) becomes $\omega \simeq \omega_{pi}$. The growth rate is obtained by reducing the expressions (24), (25), (27), (28), and (35)-(37) to this limit which by substitution into Eq. (34) gives

$$\gamma_B = \frac{1}{2}\, k_0 V_0\, \frac{\omega_p^2}{\Omega_e^2} \left(\frac{\omega_{pi}}{\omega_0}\right)^{1/2} . \tag{49}$$

The ratio between the growth rates applicable to laser fusion and electron cyclotron heating is given by the ratio of the expressions (40) and (49), respectively, which is $(\omega_p/\Omega_e)^3$.

For the stimulated Raman scattering instability the growth rate is the same as that given by Eq. (43). The ratio between this growth rate and the one for an unmagnetized plasma is given by Eq. (45) which is $2^{-1/4}$ for toroidal plasma parameters.

IV. DAMPING AND THRESHOLDS

The collisionless damping for upper hybrid waves is given by

$$\gamma_L = \frac{\pi^{1/2}}{2}\frac{1}{k^2\lambda_D^2}\frac{\omega_p^2+k^2V_e^2}{k_\parallel V_e}\left\{\exp\left(-\frac{\omega_k^2}{k_\parallel^2 V_e^2}\right)\left(1-\frac{1}{2}k_\perp^2\rho_e^2\right)\right.$$
$$\left.+\frac{1}{2}k_\perp^2\rho_e^2\left[\exp\left(-\frac{(\omega_k-\Omega_e)^2}{k_\parallel^2 V_e^2}\right)+\exp\left(-\frac{(\omega_k+\Omega_e)^2}{k_\parallel^2 V_e^2}\right)\right]\right\} \tag{50}$$

where $\omega_k^2=\omega_p^2+\Omega_e^2+k^2V_e^2$. For scattering instability in the plane perpendicular to the magnetic field, $k_\parallel$ vanishes and, consequently, the damping is greatly reduced. In the case of unmagnetized plasmas,[4] however, the Langmuir waves become heavily Landau damped for $k\lambda_D>0.2$ $(\lambda_D=V_e/\omega_p)$ and the Raman scattering instability is turned off, although, the nonlinear Compton scattering of the heat mode still persists with a smaller growth rate. But for magnetized plasmas, Raman mode prevails over nonlinear Compton scattering even for large $k\lambda_D$.

If we assume a weakly inhomogeneous plasma, the Rosenbluth threshold condition[34]

$$\frac{\gamma_0^2}{V_1V_2K'} > 1 \tag{51}$$

should hold where γ_0 is the growth rate in a homogeneous plasma, V_1 and V_2 are the components of the group velocities of the decay waves along the density gradient, and $K'=d/dx\sum(k_{0x}-k_{1x}-k_{2x})$ is the derivative in relation to the spatial variable along the direction of the density gradient. For the Brillouin scattering instability, the group velocity of the lower hybrid wave is given by

$$V_{g_{\ell h}} \simeq \left(-\frac{k_xk_z}{k^2}\hat{x}-\frac{k_yk_z}{k^2}\hat{y}+\hat{z}\right)\frac{\omega_{pi}k_z\frac{M}{m}}{k^2\left(1+\frac{k_z^2}{k^2}\frac{M}{m}\right)^{1/2}} \,. \tag{52}$$

Therefore, for scattering process in the plane perpendicular to the magnetic field the group velocity vanishes and from Eq. (51) we conclude that there is no inhomogeneity threshold for this process. For the Raman scattering process, the group velocity of an upper hybrid wave is given by

$$V_{g_{uh}} \simeq \frac{\omega_p^2}{2\omega_{uh} k' L} \tag{53}$$

which substituted into Eq. (51) together with the Raman homogeneous growth rate for magnetized plasma, gives the same threshold as in the unmagnetized case

$$\frac{1}{2}\left(\frac{V_0}{c}\right)^2 k_0 L > 1 \tag{54}$$

where $dK/dx = dk/dx$ for Raman scattering.

V. DISCUSSION AND CONCLUSION

In the fluid theory limit, the growth rates for Brillouin scattering process in magnetized plasmas are reduced in relation to the growth rates for unmagnetized plasma. For plasmas produced by CO_2 lasers, the growth rate is reduced by a factor $(k\rho_e)^{1/2}$ while for electron cyclotron heating of toroidal plasmas by a factor $(k\rho_e)^{1/2} \times (\omega_p/\Omega_e)^3$. The Raman growth rates are practically unmodified as compared to the unmagnetized plasma either for laser fusion or electron cyclotron heating parameters.

The collisionless damping is vanishingly small for stimulated Raman scattering process in a plane perpendicular to the magnetic field and, consequently, Raman instability is the prevailing process over the nonlinear Compton scattering for all values of $k\lambda_D$.

By assuming a weak inhomogeneity in the plasma density, we find that there is no threshold for the stimulated Brillouin scattering instability in a plane perpendicular to the magnetic field. For Raman scattering the threshold is the same as that in an unmagnetized plasma.

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